Math 10A
Worksheet, Discussion #32; Tuesday, 7/31/2018
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1 Determinants and Inverses

1.1 Concepts

1. The **determinant** is defined only for square matrices. It is a scalar. The determinant for a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the number $ad - bc$.

   The **inverse** of a matrix is defined only for square matrices. The inverse of A is a matrix B such that $AB = BA = I$, the identity matrix with 1s on the diagonal and 0 everywhere else. The inverse is always unique if it exists. The inverse of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the matrix $B = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. The inverse of a square matrix exists if and only if the determinant is nonzero.

   We can use the inverse to easily solve equations of the form $Ax = b$ where x, b are vectors and A is a matrix because we can write $x = A^{-1}b$ if A is invertible. This always has a **unique** solution if A is invertible. If A is not invertible, this is 0 solutions or ∞ solutions.

   There are 2 ways to calculate the determinant of a 3×3 matrix $\begin{pmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{pmatrix}$. The first is expansion along the first row to get the determinant is $b_1 \begin{vmatrix} b_5 & b_6 \\ b_8 & b_9 \end{vmatrix} - b_2 \begin{vmatrix} b_4 & b_6 \\ b_7 & b_9 \end{vmatrix} + b_3 \begin{vmatrix} b_4 & b_5 \\ b_7 & b_8 \end{vmatrix}$. The other is to use the diagonal method to get $b_1b_5b_9 + b_2b_6b_7 + b_3b_4b_8 - b_1b_6b_8 - b_2b_4b_9 - b_3b_5b_7$.

   We can determine the number of solutions to an equation $A\vec{x} = \vec{b}$ by the determinant of A and that is given below.

$\det(A)$	$\neq 0$	$= 0$
Number of Solutions	1	0 or ∞

1.2 Problems

2. True False We can take determinants of 2×3 matrices but just haven't learned it yet.

3. True False If A is a noninvertible square matrix, then $Ax = b$ may still have a unique solution.

4. True False It is possible for $A\vec{x} = \vec{0}$ to have no solutions.

5. True False If we know that $A\vec{x} = \vec{b}$ has no solutions, then we know what $\det(A)$ is.

6. True False If $\det(A) = 0$, then $Ax = b$ has no solutions.

7. Give a 2×2 matrix with determinant equal to 5. Is it unique?

8. Find the inverses for the following matrices:

$$\begin{pmatrix} 3 & 5 \\ -4 & -8 \end{pmatrix} \qquad \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} \qquad \begin{pmatrix} 1 & 5 \\ -1 & -8 \end{pmatrix}$$

9. Find the determinant of $\begin{vmatrix} 3 & 2 & 3 \\ -1 & 5 & -3 \\ 7 & -1 & -1 \end{vmatrix}$.

10. Let $A = \begin{pmatrix} 2 & 4 & -1 \\ 2 & 2 & 5 \\ 6 & 8 & 9 \end{pmatrix}$. What is $\det(A)$? How many solutions that $A\vec{x} = \vec{0}$ have?

11. Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 5 & 1 & 3 \\ 2 & 2 & 4 \end{pmatrix}$. What is $\det(A)$? How many solutions does $A\vec{x} = \vec{0}$ have?

12. Find x, y such that $2x + 3y = 4$ and $x + y = 1$.

13. Find the solution to $x + 2y = 3$ and $4x + 5y = 6$ using matrix vector form.

14. Write $x + y = 10, y + z = 5, x + z = -1$ in matrix vector form. How many solutions does it have?

15. Find a matrix X such that $\begin{pmatrix} 5 & 13 \\ 3 & 8 \end{pmatrix} X = \begin{pmatrix} 1 & 4 & 1 \\ -1 & 2 & 1 \end{pmatrix}$.